## 2006 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

3. An object moving along a curve in the $xy$-plane is at position $(x(t), y(t))$ at time $t$, where

$$\frac{dx}{dt} = \sin^{-1}\left(1 - 2e^{-t}\right) \text{ and } \frac{dy}{dt} = \frac{4t}{1 + t^3}$$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(a) Find the acceleration vector and the speed of the object at time $t = 2$.

(b) The curve has a vertical tangent line at one point. At what time $t$ is the object at this point?

(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of $t$ and use it to evaluate $\lim\limits_{t \to \infty} m(t)$.

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value $c$.

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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

## 2007 AP® CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

2. An object moving along a curve in the $xy$-plane is at position $(x(t), y(t))$ at time $t$ with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1 + t}\right) \text{ and } \frac{dy}{dt} = \ln\left(t^2 + 1\right)$$

for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the speed of the object at time $t = 4$.

(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.

(c) Find $x(4)$.

(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time $t$ is the object at this point? Find the acceleration vector at this point.

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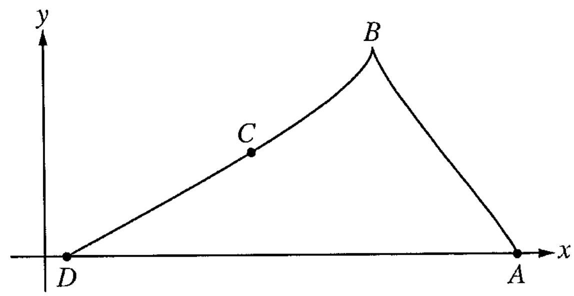
**WRITE ALL WORK IN THE EXAM BOOKLET.**




2. A particle starts at point $A$ on the positive $x$-axis at time $t = 0$ and travels along the curve from $A$ to $B$ to $C$ to $D$, as shown above. The coordinates of the particle's position $(x(t), y(t))$ are differentiable functions of $t$, where $x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)$ and $y'(t) = \frac{dy}{dt}$ is not explicitly given. At time $t = 9$, the particle reaches its final position at point $D$ on the positive $x$-axis.

(a) At point $C$, is $\frac{dy}{dt}$ positive? At point $C$, is $\frac{dx}{dt}$ positive? Give a reason for each answer.

(b) The slope of the curve is undefined at point $B$. At what time $t$ is the particle at point $B$ ?

(c) The line tangent to the curve at the point $(x(8), y(8))$ has equation $y = \frac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.

(d) How far apart are points $A$ and $D$, the initial and final positions, respectively, of the particle?



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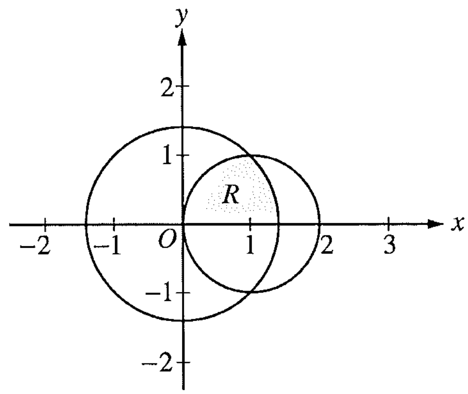


2. The figure above shows the graphs of the circles $x^2 + y^2 = 2$ and $(x-1)^2 + y^2 = 1$. The graphs intersect at the points $(1, 1)$ and $(1, -1)$. Let $R$ be the shaded region in the first quadrant bounded by the two circles and the $x$-axis.

   (a) Set up an expression involving one or more integrals with respect to $x$ that represents the area of $R$.

   (b) Set up an expression involving one or more integrals with respect to $y$ that represents the area of $R$.

   (c) The polar equations of the circles are $r = \sqrt{2}$ and $r = 2\cos\theta$, respectively. Set up an expression involving one or more integrals with respect to the polar angle $\theta$ that represents the area of $R$.

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| Distance $x$ (mm) | 0 | 60 | 120 | 180 | 240 | 300 | 360 |
|---|---|---|---|---|---|---|---|
| Diameter $B(x)$ (mm) | 24 | 30 | 28 | 30 | 26 | 24 | 26 |

3. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where $x$ represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

   (a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

   (b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

   (c) Using correct units, explain the meaning of $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

   (d) Explain why there must be at least one value $x$, for $0 < x < 360$, such that $B''(x) = 0$.

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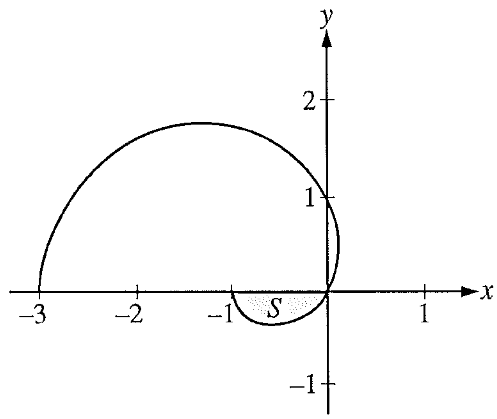
**END OF PART A OF SECTION II**

**CALCULUS BC**
**SECTION II, Part B**
**Time—45 minutes**
**Number of problems—3**

**No calculator is allowed for these problems.**




4. The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \le \theta \le \pi$ is shown above. Let $S$ be the shaded region in the third quadrant bounded by the curve and the $x$-axis.
   (a) Write an integral expression for the area of $S$.
   (b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of $\theta$.
   (c) Write an equation in terms of $x$ and $y$ for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.

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**WRITE ALL WORK IN THE EXAM BOOKLET.**